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ABSTRACT

Generalizability or invariance procedures have been known for over three decades. Through the years, these procedures have not been widely discussed or employed. One reason for the lack of use is that most of the articles on invariance procedures have been mathematically oriented. The mathematical orientation of research articles and the lack of mathematical background on the part of educational researchers have inhibited the use of generalizability procedures. Generalizability procedures are important in educational research due to the frequency of researchers' use of small sample sizes. Problems associated with small samples are compounded by the frequent use of statistical techniques that capitalize on chance and the greater popularity of multivariate techniques. This paper briefly discusses the history of three invariance procedures: (1) cross-validation; (2) jackknifing; and (3) bootstrapping. A discussion of how to use each procedure and an example of each procedure are presented. Three multivariate techniques--canonical correlation, discriminant analysis, and factor analysis--are used for the examples. These same procedures could also be employed with univariate statistics. A 33-item list of references and 13 data tables are included. The computer program used is appended. (Author/TJH)

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ESTIMATING THE GENERALIZABILITY OF MULTIVARIATE RESULTS

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ABSTRACT

Generalizability or invariance procedures have been known for over three decades. Through the years these procedures have not been widely discussed or employed. One reason for the lack of use is that the majority of the articles on invariance procedures have been mathematically oriented. The mathematical orientations of articles and the lack of mathematical background on the part of educational researchers have inhibited the use of generalizability procedures.

Generalizability procedures are important in educational research due to the frequency of researchers needing to use small sample sizes. The use of small samples is compounded by the frequent use of statistical techniques which capitalize on chance, and the greater popularity of multivariate techniques recently.

This paper briefly discusses the history of three invariance procedures: cross-validation, jackknife, and bootstrap. A discussion of how to use the procedure and an example of each of the procedures are presented. Three multivariate techniques, canonical correlation, discriminant analysis, and factor analysis, are used for the examples. These same procedures could also be employed with univariate statistics.

Estimating the Generalizability of Multivariate Results

The generalizability of research results have been under question for several years. Generally, a researcher wishes to generalize to the population from which a sample was drawn. But, results are open to skepticism if the sample may not be representative of the population or when the analytic methods employed require larger sample sizes than the research can utilize (Thompson, 1988). In addition, questions can be raised as to the reproducibility of the results because of the analytical methods used. But selected analytic methods can help resolve questions regarding the generalizability of results.

The paper discusses reasons why these generalizability analytic procedures are important. Also, three common generalizability procedures are reviewed. Concrete examples are provided using multivariate statistics to illustrate the uses of bootstrap and cross-validation.

Sample Representativeness

The first reason to use invariance procedures is to determine bounds for the results. Sample size is an important aspect to be considered for all parametric statistics, and even more so for multivariate statistics. Multiple dependent variables and their reliability increases a larger sample size for multivariate statistics. As variables, both independent and dependent, are added to the equation, there is more question about the reliability of the results. A large sample is needed to off-set the question of reliability.

A small sample may not truly represent the population. Or, the sample may introduce questions about the reliability of the results due only to sample size. Thus, sample size is one design aspect that will determine if invariance procedures are important. Invariance procedures can aid in establishing some confidence and generalizability regarding obtained results.

When a small sample is used, the two invariance procedures recommended are bootstrap and jackknife. Bootstrap, which is a procedures that concatenates the data set, and jackknife, which divides the actual data set into multiple data sets, would be recommended. These procedures are recommended since the size of the data set is not significantly reduced by the methods.

The use of multiple invariance procedures in a given study is also recommended. Thompson (1984) stated that two procedures would provide with a range in with the results would probably fall within. A confidence level would be created. The use of two procedures is not only helpful when the researcher has a small sample size, but also when correlational statistics (i.e., factor analysis, canonical correlation, Pearson correlation, OVA procedures) are employed (Thompson, 1984). Statistical methods tend to capitalize on chance occurrences and would provide an inflated result (Nunnally, 1967; Thompson, 1984). The capitalization is greatest with the OVA techniques, since data is lost often due to reducing interval data to nominal data. The invariance procedures would establish some bounds to guide interpretation.

Analytic Methods Requirements

Generalizability procedures are needed with some analytic methods more so than others. The multivariate statistical methods tend to capitalize on sampling error even more so than the univariate methods, since these methods are more sensitive to variation (Fornell, 1978). Multivariate statistics tend to be more sensitive to outliers, more so than univariate statistics. In addition to the type of analytic procedure the sample size needed for the analysis leads many researchers to use generalizability procedures. Most specifically, multivariate procedures tend to require larger sample sizes to provide more stability because of increased reliability variance and the sensitivity to variation (Balian, 1982; Katzell, 1951). Researchers may not have the luxury of a large sample and thus can rely on generalizability procedures to increase confidence in their interpretations.

Basis for Examples

The first data set is used for the cross-validation and bootstrap examples. A sample of 30 college student questionnaires were used for this data set. Student grades for their course were used as one variable. Student usage of the university resource center was recorded on the center's database, and was used as another variable. The other variables were: student attitudes about the services provided by the university's resource center, the answers on the 10 item questionnaire, and the type of class the students are enrolled in. Table 1 presents the data set.

INSERT TABLE 1 ABOUT HERE

The second data set is used only with the jackknife procedure. This data set involves data from 69 scores from a questionnaire given to teachers to determine how they regarded a typical beginning on professionalism (Daniel & OKeafor, 1987). Table 2 presents the data set. Four scores are presented: avoidance for beginning teachers, overlooking for beginning teachers, professionalism for beginning teachers, and work autonomy. Additionally the type of teacher, either experienced teachers or beginning teachers was included.

INSERT TABLE 2 ABOUT HERE

Three Invariance Procedures

Generalizability methods are frequently termed invariance procedures. There are three common methods to determine reproducibility or generalizability: (a) cross-validation, (b) Jackknife, and (c) bootstrap. Each of these techniques introduce a different type of sampling logic.

Cross-validation

The cross-validation procedure has been available to researchers prior to 1951. Mosier (1951) and Katzell (1951) discussed aspects of cross-validation. The method of cross-validation discussed in this paper Mosier

termed double cross-validation. Katzell (1951) argued that cross-validation was not used widely or recognized in the literature. Krus and Fuller (1982) stated that, after many papers, cross-validation is not frequently used.

For several years, the purpose of cross-validation seemed unclear. Horst (1966) pointed out that cross-validation does not directly assess the generalizability of the best possible equation or factor structure for a particular sample, since replication would be the acid test of generalizability. But cross-validation by randomly splitting the sample into subsets does at least provide some valid assessment of the equation derived (Drehmer & Morris, 1981), a workable method for estimating the degree of shrinkage (Kerlinger & Pedhazur, 1973), and is favored by correlationists (Cronbach, 1957).

The cross-validation procedure calls for the researcher to split the actual data set into random samples. Gorsuch (1983, p. 334) recommends the cross-validation process be used in factor-analytic [or other correlational] studies to

> replicate the analysis by randomly splitting the sample in half, factoring both halves and identifying the factors that match across the two samples.

Thus, the particular analytical technique employed for the entire data set is redone on the two random samples. The weights derived from the analysis of the two random samples are crossed or, in the case of a factor analysis, the factors are compared. Synthetic variables are created for the random sample using the actual weights and the crossed weights. The synthetic variables are compared using a Pearson product-moment correlation. If the

variables are "highly" correlated, then the sample is said to be invariant. Thus, the results would be generalizable. Otherwise, the results would not be deemed generalizable.

The example for the cross-validation uses the first data set with the analytic procedure canonical correlation. The canonical analysis is first run on the entire data set (See Appendix A for the programming). The data set is then randomly split in half. The canonical procedures is then run separately on the two hales of the original data set. Tables 3 shows the lambda values and Table 4 the canonical correlation coefficient for the three runs. Table 5 presents the function coefficients for the three matrices.

INSERT TABLES 3, 4, AND 5 ABOUT HERE

Next, the weights for the two splits are then crossed, as illustrated in the "second run" section of Appendix A. The synthetic variables are then compared using a Pearson correlation. Table 6 presents the synthetic variables created by the second run of the programming. The correlation matrix is presented in Table 7. Those scores which compares the same root with the group and weight cross are highlighted in bold. All but one pair of the crossed weight scores were statistically significant (root 1 and group 2: RT122 with RT121). Three other pairs were perfectly correlated. Most of the synthetic variables were highly correlated with each other. Thus, the results are invariant.

INSERT TABLE 6 AND 7 ABOUT HERE

Additionally, the various group variables were compared to the synthetic variables created from the run of the entire data set. Notice in most cases, all of the variables for the first root, regardless of the weights, are highly correlated to the first root, and likewise for the second root. This indicates that the two roots are distinct for the two subgroups as well as the entire sample.

Jackknife

The jackknife was first proposed by Quenouille (1949) and further refined by Tukey (1958). Jackknife is designed to "assess the accuracy or stability of an estimated parameter" (Ayabe, 1985, p. 445). For an unknown sampling distribution, the best estimate of the population value and the confidence interval around that value are derived (Spitzer, 1976).

The jackknife procedure is particularly useful for factor analysis (Pennell, 1972), canonical correlation (Dempster, 1966), and discriminant analysis (Miller, 1974; Mosteller & Tukey, 1969). The advantage is the derivation of a "direct assessment of a coefficient of stability" (Lindell & St. Clair, 1980, p. 751). One disadvantage is the inadequacy of the jackknife correcting for outliers, particularly with small data sets (Ayabe, 1985; Miller, 1974).

The jackknife procedure requires the actual data set to be divided into n groups of m size (Crask & Perreault, 1977). Then n numbers of analyses are run deleting one of the groups for each run. The same statistical analysis is used for the entire data set as well as every run of the groups. If the results are similar over the n iterations of the analysis

then the results are considered stable.

The example for the jackknife procedures uses the second data set. A discriminant analysis is used to demonstrate the use of the jackknife procedure in the multivariate case.

The first discriminant analysis was performed on the entire data set. The discriminant functions are presented in Table 8 labeled Total. There were 23 groups of three created from the data set. Twenty-three additional discriminant analyses were performed on the data set deleting one group, a different group, each run. Thus, 23 different subsamples were drawn from the original data set.

INSERT TABLE 8 ABOUT HERE

For each of the 23 runs, the discriminant functions were listed in Table 8. Notice that there is only one function coefficient for each independent variable since there is only one way. The discriminant functions from runs 11, 21, and 22 were reflected (multiplied by -1) to maintain consistency with the signs of the functions. Otherwise, the difference in the signs could cause problems with the calculations.

What are called pseudovalues were created for each of the discriminant function coefficients. The formula used was

$$J_{ay} = 23 \text{ (Total of y)} - 22 \text{ (ay)},$$

where a is the run and y is the dependent variable (23 for the use of all 23 sets and 22 for the use of 23-1 sets). The pseudovalues are presented in Table 9. The mean, standard deviation, and standard error of the pseudovalues are calculated for each dependent variable. The mean values

are the jackknife coefficients.

INSERT TABLE 9 ABOUT HERE

Additionally, 95% confidence intervals were based on the standard errors and are used to analyze the discriminant functions for all cases of the data set (or the TOTAL line). Notice the actual coefficients labeled TOTAL at the bottom of Table 9 are within the upperbound and lowerbound confidence intervals. Thus, the discriminant functions would be generalizable.

Bootstrap

Bootstrap is another estimator of bias and variance (Lunnenborg, 1985; Parr, 1983). Bootstrap provides a nonparametric inference which is broadly applicable across most parametric statistics (Diaconis & Efron, 1983; Efron, 1979, 1982; Efron & Gong, 1983; Lunneborg, 1985). Additionally, Efron (1980, 1982) demonstrated that the bootstrap can provide a more accurate depiction of the variability than jackknife.

The bootstrap procedures requires the researcher to copy the actual data set onto itself multiple times (Diaconis & Efron, 1983). Efron (1980) and Lunneborg (1985) recommended that the data set be copied between 100 to 1000 times onto itself. Then cases are randomly selected from this expanded data set to make samples. Each case has an equally good chance of being chosen for all samples. The researcher decides on the number of samples (usually dozens or hundreds) and runs the procedure on each sample. The results from the various runs are interpreted. A key statistic for the interpretation of the bootstrap procedure is the standard deviation of the

estimates. The standard deviations provide the researcher with an indication of how stable the estimates are across the samples (Thompson, 1988).

> If variations in samples do not have much effect on the estimates, then these standard deviations will be small and the researcher can [have] more confidence in a the belief that the results will generalize (Thompson, 1988, p. 682).

The example of the bootstrap procedure uses the first data set. A factor analysis is used to demonstrate the use of the bootstrap procedure in the multivariate case.

For the bootstrap procedure, the first run consists of the actual 30 cases performing a traditional factor analysis. Table 10 presents the rotated factor structure matrix. Performing the original factor analysis is of primary interest. But, the bootstrapping procedure provides some indication of the generalizability of the results from the full sample.

INSERT TABLE 10 ABOUT HERE

To begin the actual bootstrap procedure, a new data set was created by copying the data set onto itself 162 times resulting in a data set containing 4860 cases. With this new data set, three sequences of factor analysis were performed. Each sequence contained five data sets sampled from the enlarged data set. Each sample was individually factor analyzed resulting in a total of 15 factor analyses. The first sequence used a N=2430; the second sequence included 4860 cases; and, the third sequenced used a N=9720.

A mean factor structure matrix was computed for each sequence. Tables 11, 12, and 13 present the three average sequence matrices and the standard deviations for each coefficient. Notice that the original matrix and the other three matrices have similar structures. The same items "loaded" on the same factors. Thus, items 4, 6, 8, and 10 composed factor I; items 1 and 2 created factor II; items 3 and 5 made up factor III; and, factor IV included items 7 and 9 in all four matrices. But, also notice that there are varying degrees in the relationship of the items to the factors. Fourteen of the items do not fall within the upper and lower bounds for each item in each factor. In some cases, as with item 10 on factor I, there is an over estimate in the original matrix, where as item 10 on factor III, there is an under estimate of the coefficient in the original matrix. The original structure coefficients also fall within the range of the other three mean sequence matrices. Thus, the original result appears to be generalizable.

INSERT TABLES 11, 12, AND 13 ABOUT HERE

Summary

As discussed above, there are different sampling procedures for each invariance procedure. Likewise different formulas would be instituted given the various statistical analysis. These procedures can be performed for the univariate as well as the multivariate statistical procedures.

These three approaches represent three points of a continuum for estimating generalizability. At the most liberal end of estimating the

generalizability of results is cross-validation. Falling about in the middle is jackknife. And, the most conservative estimate of the generalizability of results is bootstrap. This is largely accounted by the sampling procedures employed.

The importance of the use of generalizability procedures is to provide evidence of stability of the results. If the results are not generalizable within the data set, it is impossible for the results to be generalibable across the population.

Cross-validation, jackknife, and bootstrap only estimate the generalizability of results. But, this estimate increases the confidence in results for the researcher and reader. Thus, the credibility of the results are less likely to be questioned, which is a by-product of a well thoughtout research design.

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Table 1

First Data Set
Used of Cross-Validation and Bootstrap Procedures

IT1	IT2	IT3	IT4	IT5	IT6	IT7	IT8	IT9	IT10	GPA	CLS	USE	GRP
1	1	1	2	3	4	5	6	8	7	2	1	30	1
2	2	3	4	5	6	7	8	9	9	3	1	10	2
1	1	1	9	2	8	3	7	5	6	1	0	09	2
5	5	5	4	7	4	9	3	7	7	2	0	15	1
1	1	4	5	6	9	9	7	6	4	1	0	27	2
1	6	7	2	9	3	8	3	9	4	2	0	50	1
4	4	3	2	5	7	3	6	3	8	4	1	36	2
1	1	9	8	7	6	5	5	8	9	2	0	23	1
5	5	2	8	2	9	2	8	8	8	3	1	42	2
2	3	1	4	3	2	5	5	9	9	2	0	14	1
4	3	5	3	2	5	3	7	3	5	2	0	69	2
6	6	5	4	3	7	3	5	7	2	2	0	17	1
4	3	5	3	2	6	3	5	3	6	4	1	37	1
8	8	6	4	8	5	5	2	8	4	3	1	32	1
1	1	5	2	5	2	4	3	6	3	2	1	27	2
3	3	7	7	2	6	6	2	7	2	2	0	03	2
1	2	8	3	7	4	8	8	4	8	2	1	07	2
2	3	6	6	8	4	7	7	4	8	2	1	12	1
2	4	9	8	6	6	5	4	8	9	3	1	05	2
1	3	6	6	5	7	7	5	7	8	2	0	17	1
2	5	7	6	5	8	4	6	3	9	4	1	03	2
2	1	3	3	4	5	6	5	9	9	1	1	08	1
5	3	8	7	9	7	3	9	3	8	2	0	33	2
1	2	5	7	6	4	7	4	6	4	2	1	21	1
1	1	6	6	5	7	7	8	9	8	3	1	11	2
1	1	3	1	1	2	7	3	6	4	0	1	23	2
1	1	7	2	2	4	3	3	3	2	0	0	15	1
1	2	4	4	3	3	3	5	3	3	0	0	02	1
1	1	4	3	3	3	2	5	2	3	1	1	09	2
2	2	4	3	5	4	4	3	1	3	1	1	19	1

IT1 - IT10 -- ITEM 1 TO ITEM 10 ON ATTITUDE QUESTIONNAIRE
GPA -- GRADE GIVEN FOR MATHEMATICS CLASS
CLS -- TYPE OF CLASS: REMEDIAL 0 COLLEGE LEVEL 1
USE -- USAGE OF THE LRC SERVICES
GRP -- SUBGROUP FOR THE CROSS-VALIDATION

Table 2

Second Data Set
Used of Jackknife Procedure

Code	Group	Begavoid	Begover	Begprof	Workaut
1	1	28	21	50	61
1	1	25	14	27	40
1	1	27	30	43	52
1	1	41	9	51	52
1	1	24	38	53	55
1	1	24	13	63	53
1	1	44	33	53	55
1	1	42	22	53	48
1	1	22	16	53	60
1	1	22	50	50	68
1	1	33	30	43	52
2	1	42	28	43	41
2	1	42	33	43	41
2	1	42	28	43	41
2	1	42	28	43	41
2	1	42	28

Table 3

Lambda Values for the Three Runs

Sample	Value	Approx. F	Hyp. DF	Error DF	F
ENTIRE	.66563	2.93405	4.00	52.00	.029
SAMPLE 1	.48375	2.40776	4.00	22.00	.080
SAMPLE 2	.66949	1.22186	4.00	22.00	.330

Table 4

Canonical Correlation Coefficients for the Cross-Validation Runs

Root No.	Eigenvalue	Pct.	Canon Coef.
ENTIRE DATA SET			
1	.50164	99.90805	.57798
2	.00046	.09195	.02148
SUBSET ONE			
1	.97613	95.49199	.70282
2	.04608	4.50801	.20988
SUBSET TWO			
1	.47236	97.02810	.56641
2	.01447	2.97190	.11942

Table 5

Canonical Function Coefficients

Variables	Root 1		Root2	
	Raw	Stand.	Raw	Stand.
ENTIRE DATA SET				
Class	1.79952	.90697	-.85965	-.43327
Usage	.03396	.52251	.05378	.85854
SUBSET 1				
Class	.83532	.45718	-1.74797	-.90265
Usage	.06763	.82242	.04847	.58941
SUBSET 2				
Class	-2.11259	-1.03084	.33876	.16530
Usage	-.02455	-.45446	-.05077	-.93990

Table 6

Synthetic Variables Created for Cross-Validation

ZCLASS	ZUSAGE	RT111	RT211	RT112	RT212	RT222	RT122	RT121	RT221
.86	.59	.88	-.43	-1.16	-.42	.	.	.	.
.86	-.71	.	.	.	.	.81	-.56	-.90	.22
-1.13	-.77	.	.	.	.	.54	1.51	-.99	.24
-1.13	-.38	-.83	.79	1.33	.17	.	.	.	.
-1.13	.40	.	.	.	.	-.56	.98	.51	-.12
-1.13	1.89	1.04	2.13	.30	-1.97	.	.	.	.
.86	.98	.	.	.	.	-.78	-1.33	1.26	-.31
-1.13	.14	-.40	1.10	1.10	-.32	.	.	.	.
.86	1.37	.	.	.	.	-1.15	-1.51	1.76	-.43
-1.13	-.45	-.88	.75	1.36	.23	.	.	.	.
-1.13	3.13	.	.	.	.	-3.13	-.26	4.00	-.98
-1.13	-.25	-.72	.87	1.27	.05	.	.	.	.
.86	1.05	1.25	-.16	-1.36	-.84	.	.	.	.
.86	.72	.99	-.35	-1.21	-.54	.	.	.	.
.86	.40	.	.	.	.	-.23	-1.07	.51	-.12
-1.13	-1.16	.	.	.	.	.91	1.69	-1.49	.36
.86	-.90	.	.	.	.	.99	-.48	-1.15	.28
.86	-.58	-.08	-1.12	-.62	.68	.	.	.	.
.86	-1.03	.	.	.	.	1.11	-.42	-1.32	.32
-1.13	-.25	-.72	.87	1.27	.05	.	.	.	.
.86	-1.16	.	.	.	.	1.23	-.36	-1.49	.36
.86	-.84	-.29	-1.27	-.51	.93	.	.	.	.
-1.13	.79	.	.	.	.	-.93	.80	1.01	-.25
.86	.01	.40	-.77	-.89	.13	.	.	.	.
.86	-.64	.	.	.	.	.74	-.59	-.82	.20
.86	.14	.	.	.	.	.01	-.95	.18	-.04
-1.13	-.38	-.83	.79	1.33	.17	.	.	.	.
-1.13	-1.23	-1.52	.29	1.72	.97	.	.	.	.
.86	-.77	.	.	.	.	.87	-.54	-.99	.24
.86	-.12	.29	-.85	-.83	.26	.	.	.	.

Table 7

Correlation Matrix of the Synthetic Variable

	RT111	RT211	RT112	RT212	RT222	RT122	RT121	RT221
RT111	1.0000 (15)							
RT211	-.2273 (15)	1.0000 (15)						
RT112	-.8588 (15)	.6941 (15)	1.0000 (15)					
RT212	-.6967 (15)	-.5403 (15)	.2307 (15)	1.0000 (15)				
RT222	. (0)	. (0)	. (0)	. (0)	1.0000 (15)			
RT122	. (0)	. (0)	. (0)	. (0)	.1356 (15)	1.0000 (15)		
RT121	. (0)	. (0)	. (0)	. (0)	-.9916 (15)	-.2624 (15)	1.0000 (15)	
RT221	. (0)	. (0)	. (0)	. (0)	.9916 (15)	.2624 (15)	-1.0000 (15)	1.0000 (15)

Note. RT referrs to the roots. The first number refers to the root (either 1 or 2). The second number refers to the subgroup. The third group refers to the coefficient's groups. Canonical coefficients comparable to thoes in Table 4 are in bold.

Table 8

Discriminant Functions for All Runs

Run	Begprof	Begavoid	Begover	Workaut
1	-0.60347	-0.57290	-0.13097	0.64802
2	-0.52736	-0.62933	-0.18397	0.55617
3	-0.55739	-0.57131	-0.23412	0.61426
4	-0.55483	-0.63843	-0.12155	0.59610
5	-0.60426	-0.55835	-0.17985	0.62990
6	-0.50241	-0.61446	-0.15953	0.61402
7	-0.60173	-0.56788	-0.12081	0.59386
8	-0.62761	-0.59185	-0.07371	0.65998
9	-0.57817	-0.58951	-0.13385	0.67620
10	-0.48953	-0.76112	-0.07174	0.57324
11	-0.56202	-0.65288	-0.03846	0.58380
12	-0.61904	-0.51667	-0.16324	0.66469
13	-0.56352	-0.63367	-0.12036	0.62478
14	-0.48628	-0.68936	-0.09165	0.60007
15	-0.55362	-0.57565	-0.18833	0.68912
16	-0.68099	-0.50325	-0.30048	0.73288
17	-0.56380	-0.61741	-0.12621	0.66124
18	-0.60061	-0.60252	-0.12607	0.56971
19	-0.54397	-0.61412	-0.10173	0.65827
20	-0.56019	-0.57175	-0.20340	0.68833
21	-0.58257	-0.64797	-0.01686	0.63713
22	-0.54390	-0.71921	0.02857	0.63527
23	-0.62969	-0.56861	-0.09940	0.60017
TOTAL	-0.57065	-0.61074	-0.12889	0.63075

Table 9

Pseudovalues Created from Discriminant Functions

Run	Begprof	Begavoid	Begover	Workaut
1	0.15139	-1.44322	-0.08313	0.25081
2	-1.52303	-0.20176	1.08287	2.27151
3	-0.86237	-1.47820	2.18617	0.99353
4	-0.91869	-0.00156	-0.29037	1.39305
5	0.16877	-1.75936	0.99223	0.64945
6	-2.07193	-0.5289	0.54519	0.99881
7	0.11311	-1.02632	-1.34285	1.44233
8	0.68247	-1.02632	-1.34285	-0.01231
9	-0.4.521	-1.07780	-0.01977	-0.36915
10	-2.35529	2.69762	-1.38619	1.89597
11	-0.76051	0.31634	-2.11835	1.66365
12	0.49393	-2.68028	0.62681	-0.11593
13	-0.72751	-0.10628	-0.31655	0.76209
14	-2.42679	1.11890	-0.94817	-0.65339
15	-0.94531	-1.38272	1.17879	-0.65339
16	1.85683	-2.97552	3.64609	-1.61611
17	-0.72135	-0.46400	-0.18785	-0.04003
18	0.08847	-0.79158	-0.19093	1.97363
19	-1.15761	-0.53638	-0.72641	0.02531
20	-0.80077	-1.46852	1.51033	-0.63601
21	-0.30841	0.20832	-2.59355	0.49039
22	-1.15915	1.77560	-3.59301	0.53131
23	0.72823	-1.53760	-0.77767	1.30351
Mean	-0.55916	-0.64769	-0.13534	0.630788
SD	1.00675	1.28707	1.53803	0.952321
SEe	0.20992	0.26837	0.32070	0.198572
95% LB	-0.97900	-1.18443	-0.77674	0.233642
95% UB	-0.13931	-0.11094	0.50606	1.027933
TOTAL	-0.57065	-0.61074	-0.12889	0.630750

Table 10

Original Factor Structure Matrix
After Varimax Rotation

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
IT1	.07539	**.92522**	-.02911	-.09947
IT2	-.03491	**.91079**	.21949	.11571
IT3	.02689	.06741	**.89996**	-.13627
IT4	**.72080**	.09126	.22917	.01508
IT5	.11121	.12448	**.80770**	.31875
IT6	**.80466**	.28570	-.01185	-.10737
IT7	-.11559	-.23922	.38334	**.75655**
IT8	**.81375**	-.21584	-.11394	-.05779
IT9	.08803	.16558	-.11702	**.87757**
IT10	**.68736**	-.07900	.05877	.42408

Table 11

Mean Factor Structure Matrix for the Bootstrap N of 2430

ITEM	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
IT1	.07110	.81325	-.00122	-.06788
	(.16597)	(.39453)	(.41164)	(.76902)
IT2	-.05677	.98760	.12334	.14326
	(.82746)	(.04172)	(.53981)	(.81154)
IT3	.01229	.10024	.43317	-.12637
	(.60652)	(.17539)	(.75127)	(.21330)
IT4	.73153	.03225	.26431	.00975
	(1.00234)	(.01276)	(.91848)	(.10132)
IT5	.11396	.12448	.67259	.32112
	(.77658)	(.64729)	(.62381)	(.41129)
IT6	.81259	.13267	.00121	-.10295
	(.95860)	(.59781)	(.01285)	(.43496)
IT7	-.10254	-.13426	.37571	.61473
	(.32769)	(.63817)	(.85112)	(.5879)
IT8	.80476	-.23412	-.16339	-.02786
	(.97567)	(.82653)	(.73622)	(.65862)
IT9	.06508	.12317	-.13578	.80543
	(.40922)	(.38919)	(.62871)	(.66594)
IT10	.48612	-.05231	.08562	.23192
	(.73419)	(.62385)	(.26728)	(.79527)

Note. Standard deviations in parentheses.

Table 12

Mean Factor Structure Matrix for the Bootstrap N of 4860

ITEM	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
IT1	.12485 (.65924)	.52411 (.84633)	-.09153 (.18549)	-.03278 (.71184)
IT2	-.09821 (.36593)	.65184 (.93462)	.17519 (.23386)	.19735 (.42841)
IT3	.07549 (.75407)	.03246 (.53852)	.31162 (.85590)	-.24764 (1.53391)
IT4	.68735 (.95397)	.07649 (.43628)	.21684 (.29045)	.05269 (.41922)
IT5	.16325 (.59720)	.08742 (.72043)	.56421 (.06927)	.31875 (.59831)
IT6	.64241 (.94538)	.11531 (.55936)	.00963 (.36592)	-.09863 (.65984)
IT7	-.24538 (.67758)	-.12947 (.66493)	.41764 (.19765)	.75492 (.77016)
IT8	.89861 (.76947)	-.19637 (.38892)	-.12424 (.29760)	-.05221 (.83521)
IT9	.02547 (.65836)	.09745 (.82435)	-.12676 (.19650)	.76536 (.76942)
IT10	.67346 (.56783)	-.09263 (.02785)	.11678 (.62328)	.58724 (1.03539)

Note. Standard deviations in parentheses.

Table 13

Mean Factor Structure Matrix for the Bootstrap N of 9720

ITEM	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4
IT1	.34255 (.73468)	.91127 (.64648)	-.08726 (.54836)	-.10402 (.79402)
IT2	-.12946 (.30893)	.45929 (.18659)	.12176 (.54935)	.26453 (.69264)
IT3	.09841 (.49364)	.07483 (.97347)	.59218 (.64943)	-.15660 (.46937)
IT4	.93654 (.65936)	.12848 (.29075)	.28642 (.06497)	.07437 (.29370)
IT5	.25289 (.27497)	.11659 (.74568)	.83140 (.35965)	.26461 (.54937)
IT6	.73512 (.48659)	.13726 (.29769)	.07385 (.49758)	-.15794 (.49043)
IT7	-.17383 (.34524)	-.37412 (.60648)	.29548 (.31880)	.82706 (.72038)
IT8	.91510 (.53291)	-.12473 (.04923)	-.07249 (.54939)	-.08264 (.76999)
IT9	.10736 (.75059)	.05070 (.63249)	-.27932 (.54883)	.80104 (.54979)
IT10	.59836 (.14625)	-.13735 (.79930)	.18244 (.84382)	.26917 (.54839)

Note. Standard deviations in parentheses.

Appendix A
Programming

First Run

```
TITLE 'FACTOR BOOTSTRAP & CANONICAL CROSS-VALIDATION'
FILE HALDLE BOOT/NAME='BOOT.DAT'
DATA LIST FILE=BOOT/IT1-IT10 1-10 GPA 11 CLASS 12 USAGE 13-14
     GRP 15
COMPUTE    ATTITUDE=IT1+IT2+IT3+IT4+IT5+IT6+IT7+IT8+IT9+IT10
FREQUENCIES VARIABLES=CLASS USAGE/STATISTICS=DEFAULT
MANOVA GPA ATTITUDE WITH CLASS USAGE/
       PRINT=HOMOGENEIT (BOXM)
       SIGNIF (MULTIV, EIGEN, UNIV, HYPOTH)
       DISCRIM (RAW, STAN, COR, ALPHA (.95))
       DESIGN (OVERALL, DECOMP, SOLUTION)
       OMEANS
       ERROR (SSCP, COV, COR)/DESIGN/
TEMPORARY
SELECT IF GRP=1
MANOVA GPA ATTITUDE WITH CLASS USAGE/
       PRINT=HOMOGENEIT (BOXM)
       SIGNIF (MULTIV, EIGEN, UNIV, HYPOTH)
       DISCRIM (RAW, STAN, COR, ALPHA (.95))
       DESIGN (OVERALL, DECOMP, SOLUTION)
       OMEANS
       ERROR (SSCP, COV, COR)/DESIGN/
TEMPORARY
SELECT IF GRP=2
MANOVA GPA ATTITUDE WITH CLASS USAGE/
       PRINT=HOMOGENEIT (BOXM)
       SIGNIF (MULTIV, EIGEN, UNIV, HYPOTH)
       DISCRIM (RAW, STAN, COR, ALPHA (.95))
       DESIGN (OVERALL, DECOMP, SOLUTION)
       OMEANS
       ERROR (SSCP, COV, COR)/DESIGN/
FINISH
```

Second Run

```
TITLE 'SECOND RUN FOR CROSS-VALIDATION'
FILE HALDLE BOOT/NAME='BOOT.DAT'
DATA LIST FILE=BOOT/IT1-IT10 1-10 GPA 11 CLASS 12 USAGE 13-14
   GRP 15
COMPUTE ZCLASS = (CLASS - .567)/(.504)
COMPUTE ZUSAGE = (USAGE - 20.867)/(15.391)
DO IF GRP=1
COMPUTE RT111 = (ZCLASS *   .45718) + (ZUSAGE *  .82242)
COMPUTE RT211 = (ZCLASS *  -.90265) + (ZUSAGE *  .58941)
COMPUTE RT112 = (ZCLASS * -1.03084) + (ZUSAGE * -.45446)
COMPUTE RT212 = (ZCLASS *   .16530) + (ZUSAGE * -.93990)
END IF
DO IF GRP=2
COMPUTE RT122 = (ZCLASS * -1.03084) + (ZUSAGE * -.45446)
COMPUTE RT222 = (ZCLASS *   .16530) + (ZUSAGE * -.93990)
COMPUTE RT121 = (ZUSAGE *   .45718) + (ZUSAGE *  .82242)
COMPUTE RT221 = (ZUSAGE *  -.90265) + (ZUSAGE *  .58941)
END IF
PEARSON CORR RT111 TO RT221
FINISH
```